CAMBRIDGE ASSESSMENT

STEP Examiners' Report 2012

Mathematics
STEP 9465/9470/9475

November 2012

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STEP Mathematics (9465, 9470, 9475)

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Comments on individual questions

Q1

This was a popular question and most candidates managed to find expressions for p and q easily. Many did not make any progress beyond this point however. Of those that continued with the question many managed to find the minimum value of $p+q$ easily, but the minimum value of $\sqrt{p^2+q^2}$ caused more trouble. The majority of candidates did not realise that they could simply differentiate p^2+q^2 to find the minimum value and so attempted a more complicated differentiation. In many cases they were successful, but then failed to find the value of m to complete the question.

Q2

This was again a popular question with candidates achieving varying levels of success. The first part of the question was generally well attempted, although a number of candidates who identified that the equation could be rewritten as $y=(x^2-3)^2$ assumed that this would result in a sharp change of gradient at the points of intersection with the x -axis (as would be seen in the graph of $y=|x^2-3|$).

Some candidates attempted to regard the equation as a quadratic in x^2 to identify the number of roots for different cases, but this did not sufficiently distinguish between the different cases in the solutions provided. A better method is to consider the intersections of two graphs as the value of b is varied. Another common mistake was to give just part of the solution set in the case $n=2$ or to assume that it is not possible for there to be three solutions.

In the second part of the question most candidates correctly found the two possible values of a , but often just chose one of the values to work with for the next part. Graphically, the difference between the two cases is a reflection in the y -axis and, since the intersections with a horizontal line are being found, the sets of values of b will be the same in both cases. This was not explained in choices that decided to explore just one of the two cases.

Q3

The first part of this question was generally well answered by those candidates who recognised the concept of placing the required area between two triangles. There were a number of answers where the graph was sketched, but no further progress was made however.

Of those who solved the first part of the question, many were able to do some work on the second part of the question, but took the upper limit of the integral as $x=a$ (consistent with the method for the first part of the question, but not required here as $x=1$ is the upper limit to choose) and therefore arrived at far more complicated expressions than were needed. Often this led to a doomed attempt to handle the algebra that was left.

For those who correctly chose the limits of the integration the part of the inequality whose solution involved $(\ln a)^2$ still proved a challenge for some.

Q4

This question was generally well attempted, with many candidates able to obtain the equations of the tangents and normals. The point of intersection of the tangents did not cause too much difficulty, but the intersection of the normal was problematic for many candidates who struggled to simplify the expressions and therefore did not reach a point where it was obvious that the point lay on the line $y = x$.

The final part of the question is much easier if the expressions are simplified as they are encountered and for this the factorisation of the difference of two cubes is a useful thing to know. It is also useful to realise that the information that $2pq = 1$ means that the expression $p^2 + q^2 + 1$ can be factorised as $(p + q)^2$.

Q5

This was another popular question, with attempts by a large number of candidates. Integration by parts was required for both of the main methods for the first integral, either preceded or followed by the application of some trigonometric identities. A number of candidates managed to obtain the correct answers for each of the first two integrals, but then struggled to relate them to the final part of the question, in some cases ignoring the different limits and in others incorrectly manipulating the logarithms.

Q6

This question was not attempted by many candidates, possibly due to the apparent three-dimensional nature of the problem (although it reduces to a two-dimensional problem immediately). Many candidates solved the first part of the problem through applications of Pythagoras theorem in the various right-angled triangles that can be identified rather than using the similarity that is present. A number of candidates got the expressions for the tangent and cotangent confused.

Where candidates attempted the second part they were generally more successful, although care needed to be taken over the individual steps. Relating the identity to the first part of the question involved an understanding of the trigonometric graphs and this was done successfully by a number of candidates.

Q7

The first two parts of this question were generally well answered, although a number of candidates were confused about the order in which to substitute the variables into the equation and thus got answers with p and q confused. The third part proved more complex, with the factorisation of the final expression causing problems for some candidates.

Q8

This was a very popular question with attempts by most of the candidates. In many cases the first substitution was correctly carried out and the resulting differential equation solved, but then no progress was made on the second part.

The candidates who realised that the same substitution would work for the second part managed to get to the reduced differential equation. Although the integration for this differential equation was more complicated, many of the candidates who reached this stage managed to evaluate the integral (although sometimes by longer methods than were needed).

Q9

This question was quite popular. The first part of the question involves the application of the formulae for motion under uniform acceleration and this was generally well carried out, although a number of candidates did not justify the choice of the positive square root. A number of candidates also took a longer approach to the calculation, calculating the time to reach the highest point and then the time for the downward journey. Many candidates realised that differentiation of the expression for the range was required, although some decided to differentiate with respect to θ , rather than c , making the task more difficult. The differentiation requires a degree of care to make sure that the signs are correctly managed and many candidates did manage to complete this successfully.

Q10

For many candidates the first part of the question was solved correctly. The manipulation of the expressions involving sums and differences of square roots was more complicated for a number of candidates and the derivation of the formula required in the second part was less successfully carried out. The final part of the question involved the substitution of the values given into the formula, and providing that this was done with care the correct answer was generally found successfully.

Q11

This question began with some quite familiar calculations involving inclined planes and many candidates who attempted the question were able to reach the solution. In some cases the required value of M was assumed rather than solving the simultaneous equations.

The second part of the question was less familiar and some candidates did not realise that the fact that the pulley can move means that there will be different accelerations at different points in the system. They therefore attempted to calculate one value for the acceleration that worked for all of their equations.

Q12

This question was not attempted by many candidates. In some cases it was not understood that the function had to be integrated to find the probabilities. The identification of the probabilities required for the conditional probability calculation was also problematic. Where it was there were some good answers, although the algebraic manipulation proved a little complicated for some candidates.

Q13

This question was not attempted by many candidates. There were some good answers showing a clear thought process to reach the required value, but many of the other solutions offered suffered from a lack of explanation of the method meaning that the ideas being applied were difficult to follow.

General Remarks
There were just over 1000 entries for paper II this year, almost exactly the same number as last year. Overall, the paper was found marginally easier than its predecessor, which means that it was pitched at exactly the level intended and produced the hoped-for outcomes. Almost 50 candidates scored 100 marks or more, with more than 400 gaining at least half marks on the paper. At the lower end of the scale, around a quarter of the entry failed to score more than 40 marks. It was pleasing to note that the advice of recent years, encouraging students not to make attempts at lots of early parts to questions but rather to spend their time getting to grips with the six that can count towards their paper total, was more obviously being heeded in 2012 than I can recall being the case previously.

As in previous years, the pure maths questions provided the bulk of candidates' work, with relatively few efforts to be found at the applied ones. Questions 1 and 2 were the most popular questions, although each drew only around 800 "hits" – fewer than usual. Questions 3 – 5 & 8 were almost as popular (around 700), with Q6 attracting the interest of under 450 candidates and Q7 under 200. Q9 was the most popular applied question – and, as it turned out, the most successfully attempted question on the paper – with very little interest shown in the rest of Sections B or C.

Comments on individual questions

Q1 The first question is set with the intention that everyone should be able to attempt it, but 20% of candidates were clearly put off by the algebraic nature of this year's opener. It was , nonetheless, the most popular question on the paper, possibly due to the lack of any advanced techniques. The specifically numerical parts of the question were generally more confidently, and hence successfully, handled than the general ones. So, for instance, formulae for the various coefficients in (i) were often not correct, even when high marks were scored on the question. When numerical answers went astray, it was usually due to incorrect signs in the early stages, with candidates failing to realise that all terms were positive. The very final demand (for the coefficient of x^{66}) was the real test, not only of candidates' resilience and nerve but also of their grasp of where the various contributions were coming from. From a marking point of view, very few candidates gave particularly clear methods, and it was usually difficult for the markers to decipher the underlying processes from what appeared to be merely a whole load of numbers written down and added up.

Q2 This turned out to be the second most popular question and the highest scoring of the pure questions. Explanations apart, most candidates held their nerve remarkably well to produce careful algebra leading to correct answers. The added "trap" in part (i) – in that each answer contained an arbitrary constant – caught many out.

Q3 This was another popular question, scoring just over half of the marks on average. Although most efforts to establish the given initial result were eventually successful, many made hard work of it, failing to notice the obvious result that if $t = \sqrt{x^2+1}+x$ then $\frac{1}{t} = \sqrt{x^2+1}-x$.

The first integral could then be found by realising that $f(t) = \frac{1}{t^2}$ was the relevant function here, or by repeating the substitution already used.

Quite a few candidates thought that the second integral followed from the first, which was unfortunate, as it didn't. However, most efforts at this second integral were unsuccessful anyhow, with candidates usually getting 3 of the 10 marks for setting up the substitution and then often going round in circles. The main problem lay in using sin and cos instead of tan and sec, or in continuing with $\sqrt{x^2+1}+x$ without identifying a suitable function f(t). It was helpful to find this, but not essential.

Q4 This was quite a popular question, as candidates seemed to like using the log series, and appreciated the helpful structuring of the question. However, inequalities are seldom entirely confidently handled, and explanations (wherever they are required) are generally rather feeble. Thus, several marks were often not picked up, sometimes because the candidates did not think they had to consider addressing issues such as whether the series was valid in this case. Part (i) was usually fairly well done, with (ii) providing more of a challenge. Part (iii) required only informal arguments, but many scored only 1 of the 2 marks allocated here due to being a bit too vague about what was going on.

Q5 Another popular question, but scoring a relatively low average mark overall; however, this was partly due to the high number of partial attempts, and good efforts usually scored around 14 marks. Surprisingly, the greatest difficulty was found in the differentiation in part (ii). There were lots of marks for the curve-sketching, and several easy features to work with: principally the symmetry and the asymptotes (and the behaviour of the functions on either side of these). For many who struggled, the biggest problem lay in where to put the y-axis, which was largely immaterial. As mentioned already, differentiation attempts were rather poor on the whole, with muddling of the *Chain*, *Product* and *Quotient Rules*. Even for those who differentiated correctly, extracting the factor $(2x - a - b)$ proved too tough, despite the fact it might have been obvious with a bit of thought.

Q6 This was the second least popular of the pure maths questions, partly (it seems) because many candidates did not know what was meant by the term *cyclic quadrilateral*. The other immediate hurdle was that candidates needed to know that "*opposite angles of a cyclic quad. are supplementary*". We know this because of the large number of "attempts" that got no further than an initial diagram and a bit of working. Thus, the question was even less popular than the raw figures show. In reality, the question involved little more than some GCSE-level trigonometry, the *difference of two squares factorisation* and the result $\sin^2 + \cos^2 = 1$. Those who overcame the initial hurdles scored highly.

Q7 This question was the least popular of the pure maths questions by a considerable margin, and attempts at it were usually fairly poor. In fact, very few candidates got beyond the opening (given) result. The barrier to further progress was almost invariably the failure to realise that all points on a circle centre O and radius 1 have position vectors that satisfy $\mathbf{x}\bullet\mathbf{x} = 1$.

Q8 Well over a half of all candidates attempted this question but, on average, it proved to be the least well scoring. The initial inequality was usually well handled, but most of the remaining parts of the question were poorly handled in very circuitous ways, with few candidates being very clear in either what they were trying to prove or how. The first, given, result simply follows from equating for q^2 in two successive cases of the given recurrence definition. This result was then supposed to help with the following result, but almost no-one seemed to realise this, and attempts at inductive proofs were quite common at this stage (usually unsuccessfully). Candidates' confidence had clearly ebbed away well before the final part of the question, and so there were very few attempts at the two cases of the final paragraph.

Q9 Despite its obviously algebraic nature, and incorporating inequalities, this was a remarkably popular question with candidates, more than 400 of whom chose to do it. Moreover, it also proved to be the most successful question on the paper, with the average mark exceeding a score of 12. Marks that were lost generally arose from a lack of care with signs (directions) or a failure to justify the direction of the inequality from the physical nature of the situation.

Q10 This was the least popular question on the paper, attracting the poorest efforts and having the weakest mean score (under 4 marks). Around half of attempts foundered at the very outset by failing to have "the vertical plane containing the rod ... perpendicular to the axis of the cylinder". Those candidates who resolved horizontally and vertically, instead of parallel and perpendicular to the rod, invariably ended up with a terrible mess that they simply couldn't sort; a few forget to take moments at all and were thus unable to make much progress towards the answers required.

Q11 There were many very capable attempts at this question, taken by around a quarter of all candidates. At some stage, a general approach was required to the use of the principle of conservation of linear momentum; some resorted to "pattern-spotting" (which lost them a couple of marks) and others to an inductive approach, collision by collision, which worked well though was generally a lengthier bit of work. Some candidates mixed up n with N in the following part, while others incorrectly considered $>$ rather than $\geq$ and ended up missing the answer by 1. A little bit of care was needed with the summation in the final part, and there was a bit of fiddling going on in order to get the given answer. Nevertheless, it was pleasing to see the principles understood well, even if the details were less carefully attended to.

Q12 This was not a popular question, and most attempts petered out after part (i), which was usually handled very well, even when the situation was split into more cases than was strictly necessary. Indeed, few made much of a serious attempt at (ii), mainly because they were either finding the range of p such that (i)'s given answer was equal to $\frac{3}{7}$, or solving $2.5 < \mathrm{E}(X) < 3.5$, where X was the number of days that the light was on.

Q13 This question drew very little interest from candidates. Most attempts gained the first couple of answers and then differentiated to find the pdf of Y. In the attempts to find $\mathrm{E}(Y)$ and $\mathrm{E}(Y^2)$, most candidates rightly attempted *integration by parts*, although some coefficients went astray when either making a substitution or comparing the integrals with the corresponding ones of the standard normal distribution. Slightly surprisingly, it was relatively common to find $\mathrm{E}(Y^2)$ correct but the variance incorrect, as candidates failed to make this modest extra step without error.

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STEP 3 2012 Examiners' report

The number of candidates attempting more than six questions was, as last year, about 25%, though most of these extra attempts achieved little credit.

1. In spite of the printing error at the start of the question, two thirds of the candidates attempted this question. Most candidates earned a quarter of the marks by obtaining *z* in terms of *y* in part (i), and then went no further. Some candidates realised the significance of the first line of the question that the expression given was an exact differential, and those that did frequently then scored highly. Some candidates found their own way through having obtained *z* in terms of *y* in part (i), then making y the subject substituted back to find a second order differential equation for z, which they then solved and hence completed the solution to each part.

2. Three quarter s of the candidates attempted this question, making it the second most popular, and one of the two most successful. Generally, part (i) was successful attempted, though at times marks were dropped through insufficient explanation and quite a few struggled to deal with the "remainder term". Some candidates expanding the brackets worked with the second and third, the fourth brackets etc., only including the first bracket last. About half the candidates considered the product of all of the denominators in part (ii) and replicated the method for the first part, whilst others used the results from part (i), replacing x by x^3 and employing the factorisations of sums and differences of cubes. Full marks were not uncommon on this question, nor were half marks.

3. Two thirds of candidates attempted this question, but generally, with only moderate success earning just less than half marks. The vast majority of candidates (more than 85%) did not observe that, regardless of the case, the two parabolas "touch exactly once", dropping 4 or 5 marks immediately. However, most managed to obtain the three results in part (ii), though a few seemed to forget to derive that for *k*. Unaccountably, many threw away the final marks, only considering the case $a = 1$.

4. Just over 70% of the candidates attempted this, with marginally less success than question 3. Lots of attempts relied on manipulating series for e, and would have struggled had the first two results not been given, and even so, there were varying levels of success and conviction. This approach fell apart in the this part with the cubic term. Some candidates used a generating function method successfully with an $(x) = \sum_{n=1}^{\infty} \frac{n+1}{n!} x^n$. However, whilst this worked well for part (i), it got very nasty for part (ii). There were lots of sign errors with the log series in part (ii), having begun well with partial fractions.

5. This was only very slightly more popular than question 4, though with the same level of success. A lot of candidates scored just the first 5 marks, getting as far as completing the simplification in part (i) (b) , but then, being unable to apply it for the final result, and then making no progress with part (ii). The biggest problem was that candidates ignored the definitions given at the start of the question, most notably that "*a* and *b* are rational numbers". The other common problem was that candidates chose a simple value for θ such as $\frac{\pi}{4}$ or $\frac{\pi}{3}$ rather than for $\cos\theta$ such as $\frac{4}{5}$. In part (ii), quite frequently, candidates substituted $x = p + \sqrt{2}q$, and $y = r + \sqrt{2}s$ and some then successfully found solutions. For part (ii) (c), a method using $\cosh\theta$ and $\sinh\theta$ was not unexpected, although the comparable one with $\sec\theta$ and $\tan\theta$ was quite commonly used too.

6. Two thirds attempted this, with less success than its three predecessors. Very few indeed scored full marks, for even those that mastered the question rarely sketched the last locus correctly, putting in a non-existent cusp. Most candidates managed the first part, good ones the second part too, and only the very best the third part. Quite a few assumed the roots were complex and then used complex conjugates, with varying success. Many candidates lost marks through careless arithmetic and algebraic errors. Given that most could do the first part, it was possible for candidates to score reasonably if they took care and took real parts and imaginary parts correctly.

7. Two thirds attempted this too, with marginally greater success than question 2. Most did very well with the stem, though a few were unable to obtain a proper second order equation. Those that attempted part (i) were usually successful. The non-trivial exponential calculations in part (ii) caused problems for some making computation mistakes whilst others were totally on top of this. Part (iii) tested the candidates on two levels, interpreting the sigma notation correctly, and recognising and using the geometric series. Some managed this excellently.

8. This was the most popular question attempted by over 83% of candidates, and the third most successful with, on average, half marks being scored. Part (i) caused no problems, though some chose to obtain the result algebraically. Part (ii) was not well attempted, with a number stating the two values the expression can take but failing to do anything else or failing with the algebra. Part (iii) was generally fairly well done although frequently the details were not quite tied up fully.

9. The second least popular question attempted by only a couple of dozen candidates with very little success, less than any other question. The problem was none of the candidates appreciated how to handle the algebra to obtain the first result, even if they had obtained the equations of motion. Unfortunately, they rarely had the full set of equations of motion. As a consequence, they made no progress on the second part.

10. This was the most popular of the non-Pure questions, being attempted by about a sixth of candidates, and with more success than all but three questions on the paper. Many attempts failed to include a clear, legible, accurate diagram, and so an unclear mess of variables invariably failed to lead to satisfactory conclusions. On the other hand, the general standard of mechanics was above average, and the initial energy equation was usually correct. Many candidates came to grief with the general energy equation, confusing signs. A good number of strong candidates ploughed straight through correctly, and all who did so, then gained credit at the end for using the discriminant to demonstrate that R is non-zero.

11. This was slightly less popular than question 10, and slightly less success was achieved. Most candidates correctly evaluated the kinetic and potential energies of the particle, and the kinetic energy of the rope. However they had more difficulty finding the potential energy of the rope, and put themselves at an unnecessary disadvantage by not explaining their logic. There were different ways of splitting up the rope, which one they used they frequently failed to make clear, and likewise those calculating potential energy relative to a reference point failed to make the choice of that point clear. The second part of the question was done very well using the result given for the first part. The last part was fairly easy, but quite a few candidates did not justify the logic fully.

12. Under 9% of candidates attempted this, though the level of success was comparable with that achieved in questions 3 to 6, 10 and 11. The derivation of the pdf was, in many cases, the stumbling point whether being found directly, or via the cpf, lacking clear explanation. The expectation caused few problems. The second part reflected the first in each respect.

13. Even fewer attempted this than question 9. It was the second least successfully attempted question. Generally, part (i) was reasonably attempted although a number of attempts were very unconvincing as candidates failed to approach this as conditional probability. Hardly any got properly to grips with the second part, though some cashed in with the final variance result.

Explanation of Results STEP 2012

All STEP questions are marked out of 20. The mark scheme for each question is designed to reward candidates who make good progress towards a solution. A candidate reaching the correct answer will receive full marks, regardless of the method used to answer the question.

All the questions that are attempted by a student are marked. However, only the 6 best answers are used in the calculation of the final grade for the paper.

There are five grades for STEP Mathematics which are:
S – Outstanding
1 – Very Good
2 – Good
3 – Satisfactory
U – Unclassified

The rest of this document presents, for each paper, the grade boundaries (minimum scores required to achieve each grade), cumulative percentage of candidates achieving each grade, and a graph showing the score distribution (percentage of candidates on each mark).

STEP Mathematics I (9465)

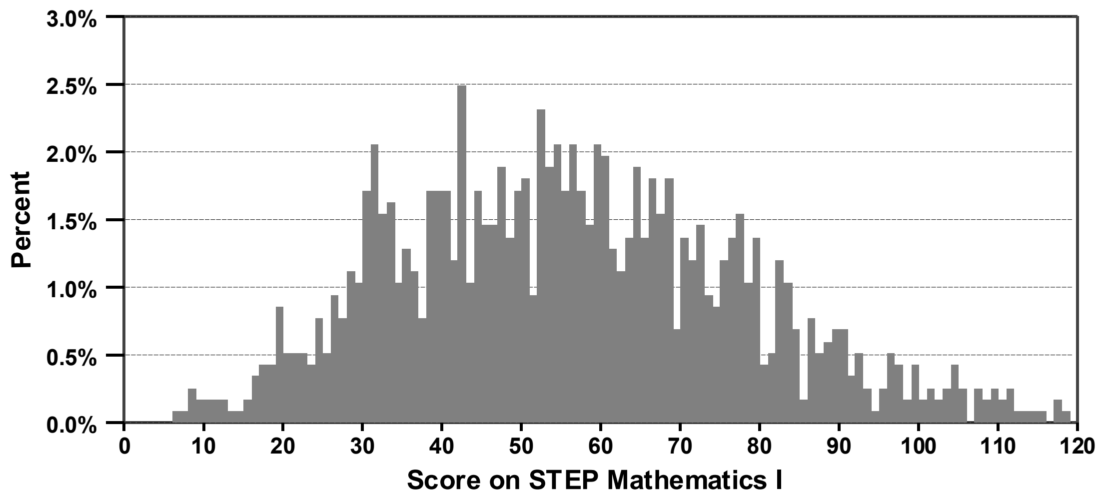
Grade boundaries

Maximum Mark	S	1	2	3	U
120	93	77	54	35	0

Cumulative percentage achieving each grade

Maximum Mark	S	1	2	3	U
120	5.9	18.9	53.7	82.4	100.0

Distribution of scores



STEP Mathematics II (9470)

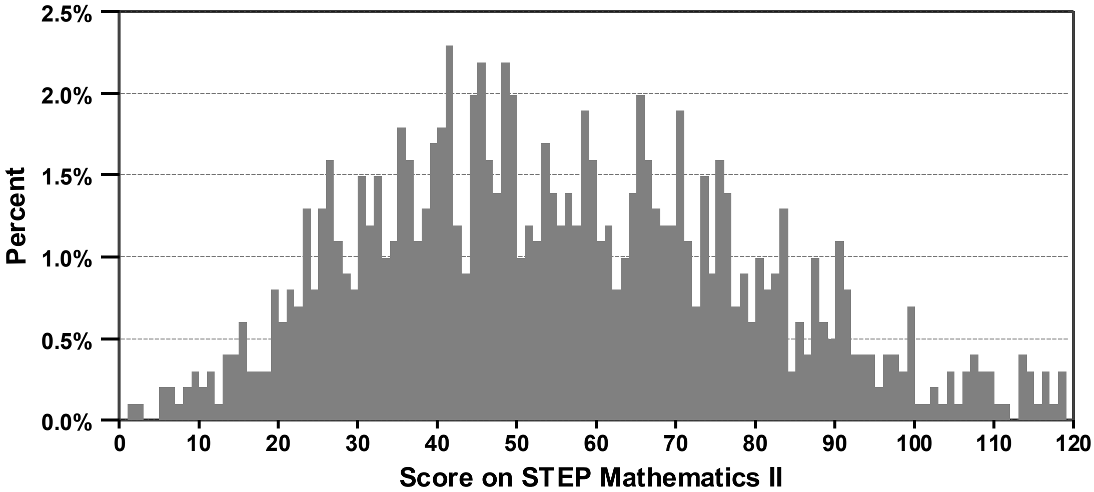
Grade boundaries

Maximum Mark	S	1	2	3	U
120	91	72	60	31	0

Cumulative percentage achieving each grade

Maximum Mark	S	1	2	3	U
120	9.0	25.7	41.9	85.2	100.0

Distribution of scores



STEP Mathematics III (9475)

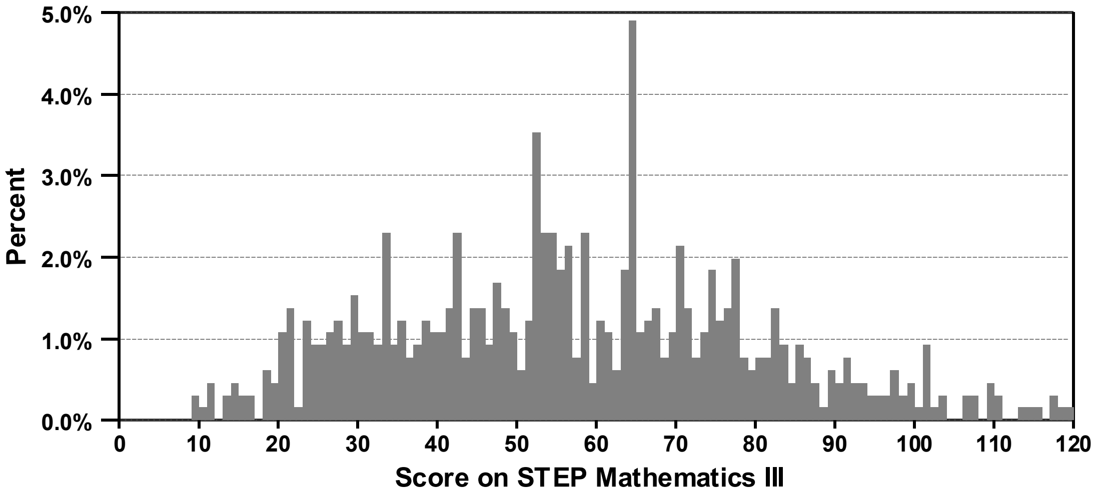
Grade boundaries

Maximum Mark	S	1	2	3	U
120	84	65	53	32	0

Cumulative percentage achieving each grade

Maximum Mark	S	1	2	3	U
120	12.7	39.2	59.6	85.2	100.0

Distribution of scores



We very much regret a printing error in the STEP Mathematics Paper III taken on 27 June, which resulted in question 1 (an optional question) being affected.

We have taken steps to identify the candidates who may have been affected and to ensure that this does not affect their applications for university places.

The mark scheme for question 1 was adjusted to reflect the change to the question, and markers were trained accordingly. After marking was complete, the scripts of all candidates who attempted question 1, together with any representations made on behalf of candidates, were considered individually by a team led by the Chief Examiner. Where a candidate was found to have been placed at a disadvantage, appropriate adjustments were made following the grading meeting.

We are undertaking a full review of our processes to make sure that errors of this kind are not repeated.